Published by Institute of Physics Publishing for SISSA/ISAS
Received: *July 29, 2004*
Accepted: *October 23, 2004*

Composite electric S-brane solutions with maximal number of branes

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Abstract: In this paper we consider $(n+1)$-dimensional cosmological model with scalar field and antisymmetric $(p+2)$-form. Using an electric composite Sp-brane ansatz the field equations for the original system reduce to the equations for a Toda-like system with $n(n-1)/2$ quadratic constraints on the charge densities. For certain odd dimensions ($D = 4m+1 = 5, 9, 13, \ldots$) and $(p+2)$-forms ($p = 2m-1 = 1, 3, 5, \ldots$) these algebraic constraints can be satisfied with the maximal number of charged branes (i.e. all the branes have non-zero charge densities). These solutions are characterized by self-dual or anti-self-dual charge density forms Q (of rank $2m$). For these algebraic solutions with the particular D, p, Q and non-exceptional dilatonic coupling constant λ we obtain general cosmological solutions to the field equations and some properties of these solutions are highlighted (e.g. Kasner-like behavior, the existence of attractor solutions). We prove the absence of maximal configurations for $p = 1$ and even D (e.g. for $D = 10$ supergravity models and those of superstring origin).

Keywords: Field Theories in Higher Dimensions, p-branes.

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1. Introduction

In this paper we investigate composite electric S-brane solutions (space-like analogues of D-branes) in an arbitrary number of dimensions D with scalar field and $(p + 2)$-form. The $(p + 2)$-form is considered using a composite electric ansatz and the metric is taken as diagonal. All ansatz functions for the metric, form field and scalar field are taken to depend on only one distinguished coordinate which is taken as time-like for the cosmological solutions considered in this paper. Previously, related work on cosmological and S-brane solutions can be found in [1]–[11] and references therein.

The procedure we use to investigate our system of multi-dimensional gravity plus scalar field plus form field is similar to the approach used in [12]. This work also studied a system with scalar fields and antisymmetric forms field defined on the manifold $M_0 \times M_1 \times \cdots \times M_n$ (M_i are Einstein spaces and $i \geq 1$). The form fields were taken in the form of a composite electro-magnetic p-brane ansatz, the metric was block-diagonal, and all scale factors and fields depended upon coordinates of M_0. Under these conditions the original model could be reduced to a gravitating, self-interacting sigma-model on M_0 with quadratic "constraints" on the charge densities. These constraints came from the non-diagonal part of the Einstein-Hilbert equations. It was shown that the constraints

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could be satisfied for certain "non-dangerous" intersection rules of the branes [12]. In the present work we use the same sigma-model approach to show that it is possible to satisfy the constraints maximally (i.e. all the branes carry non-zero charge densities) in certain odd dimensions. We then examine cosmological solutions in these odd dimensional cases, and discuss some of their interesting features such as their Kasner-like behavior.

The importance of studying solutions with "maximal" number of branes is related to research of oscillating behavior of cosmological solutions near the singularity [13, 14, 15], e.g. using the so-called billiard approach [13]. In [14] and other related works (for a review, see [15]) it was argued that in superstring cosmology one gets chaotic behavior in terms of the "oscillations" of Kasner parameters as one approaches the cosmological singularity. In these works the maximal number of electric branes were considered.

In the next two sections we will give the set up for the system of $D = n+1$ dimensional gravity, plus a scalar field, plus a $(p+2)$-form field. For the conditions considered in this paper (diagonal metric, composite Sp-brane ansatz for the antisymmetric $(p+2)$-form field, and all the ansatz functions depending only on one coordinate) this complex system can be reduced to a 1-dimensional σ-model. This transformation greatly helps in studying the solutions of the system.

In section 4 we consider the quadratic constraints for the charge densities of the branes. We find that these constraints have "maximal" solutions with all non-zero brane charge densities in particular odd dimensions with particular form fields: $D = 5, 9, 13, \ldots$ and $p = 1, 3, 5 \ldots$, respectively. We prove also the absence of maximal configurations for $p = 1$ and even D.

In section 5 we investigate cosmological solutions to the field equations for these odd dimensions. We look at the proper time behavior of the simplest of these solutions. We also show that certain solutions exhibit Kasner-like behavior at these early times.

2. D-dimensional gravity coupled to q-form and scalar field

Here as in [12] we consider the model governed by the action

$$S = \int_M d^D z \sqrt{|g|} \left[R[g] - g^{MN} \partial_M \varphi \partial_N \varphi - \frac{1}{q!} \exp(2\lambda\varphi) F^2 \right], \tag{2.1}$$

where $g = g_{MN} dz^M \otimes dz^N$ is the metric, φ is a scalar field, $\lambda \in \mathbb{R}$ is a constant dilatonic coupling and

$$F = dA = \frac{1}{q!} F_{M_1 \ldots M_q} dz^{M_1} \wedge \ldots \wedge dz^{M_q}, \tag{2.2}$$

is a q-form, $q = p + 2 \geq 1$, on a D-dimensional manifold M.

In (2.1) we denote $|g| = |\det(g_{MN})|$, and

$$F^2 = F_{M_1 \ldots M_q} F_{N_1 \ldots N_q} g^{M_1 N_1} \ldots g^{M_q N_q}. \tag{2.3}$$

The equations of motion corresponding to (2.1) are

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN} \,, \tag{2.4}$$

$$\triangle[g]\varphi - \frac{\lambda}{q!} e^{2\lambda\varphi} F^2 = 0 \,, \tag{2.5}$$

$$\nabla_{M_1}[g](e^{2\lambda\varphi} F^{M_1 \ldots M_q}) = 0 \,. \tag{2.6}$$

In (2.5) and (2.6), $\triangle[g]$ and $\bigtriangledown[g]$ are Laplace-Beltrami and covariant derivative operators corresponding to g. Equations (2.4), (2.5) and (2.6) are, respectively, the multidimensional Einstein-Hilbert equations, the "Klein-Fock-Gordon" equation for the scalar field and the "Maxwell" equations for the q-form.

The source terms in (2.4) can be split up as

$$T_{MN} = T_{MN}[\varphi, g] + e^{2\lambda\varphi} T_{MN}[F, g] \,, \tag{2.7}$$

with

$$T_{MN}[\varphi, g] \; = \; \partial_M \varphi \partial_N \varphi - \frac{1}{2} g_{MN} \partial_P \varphi \partial^P \varphi \,, \tag{2.8}$$

$$T_{MN}[F, g] \; = \; \frac{1}{q!} \left[-\frac{1}{2} g_{MN} F^2 + q F_{M M_2 \ldots M_q} F_N{}^{M_2 \ldots M_q} \right] , \tag{2.9}$$

being the stress-energy tensor of the scalar and q-form, respectively.

Let us consider the manifold

$$M = (u_-, u_+) \times \mathbb{R}^n \tag{2.10}$$

with the metric taken to be diagonal and of the form

$$g = w \mathrm{e}^{2\gamma(u)} du \otimes du + \sum_{i=1}^{n} \mathrm{e}^{2\phi^i(u)} \varepsilon_i dy^i \otimes dy^i \,, \tag{2.11}$$

where $w = \pm 1$, and u is a distinguished coordinate. The metric ansatz functions $\gamma(u), \phi^i(u)$, the scalar field $\varphi(u)$ and the q-forms are assumed to depend only on u. For definiteness one can think of u as the "time" coordinate but when $w = +1$, u is space-like. Here

$$\varepsilon_i = \pm 1 \tag{2.12}$$

are signature parameters with $i = 1, \ldots, n$. When u is time-like and all $\varepsilon_i = 1$ the solutions are cosmological. The functions $\gamma, \phi^i \colon (u_-, u_+) \to \mathbb{R}$ are smooth.

In order to deal in a general way with the different possible indices for the various forms (q-form, volume form) we define

$$\Omega_0 = \{\emptyset, \{1\}, \ldots, \{n\}, \{1,2\}, \ldots, \{1, \ldots, n\}\} \tag{2.13}$$

which is the set of all subsets of

$$I_0 \equiv \{1, \ldots, n\} \,. \tag{2.14}$$

These sets indicate the number and ranges of the indices of the q-forms.

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For any $I = \{i_1, \dots, i_k\} \in \Omega_0$ with $i_1 < \dots < i_k$, we define a form of rank $d(I) \equiv k$

$$\tau(I) \equiv dy^{i_1} \wedge \dots \wedge dy^{i_k} \,. \tag{2.15}$$

The corresponding brane submanifold is described by coordinates $y^{i_1}, \dots, y^{i_k}$. We also define the $\mathcal{E}$-symbol as

$$\mathcal{E}(I) \equiv \varepsilon_{i_1} \dots \varepsilon_{i_k} \,. \tag{2.16}$$

We adopt the following electric composite Sp-brane ansatz for the field of the $(p+2)$-form

$$F = \sum_{I \in \Omega_e} d\Phi^I \wedge \tau(I) \tag{2.17}$$

where the set

$$\Omega_e \equiv \{I \in \Omega_0 | d(I) = q - 1 = p + 1\} \tag{2.18}$$

contains all subsets of Ω_0 of the "length" $p+1$, i.e. of the form $\{i_0, i_1, \dots, i_p\}$.

We assume that the scalar potential and the scalar field only depend on the distinguished coordinate

$$\Phi^I = \Phi^I(u) \,, \qquad \varphi = \varphi(u) \,. \tag{2.19}$$

3. σ-model representation with constraints

3.1 σ-model

The system of the previous section can be greatly simplified. In [12] (see Proposition 2 in [12]) it was shown that the diagonal part of Einstein equations (2.4) and the equations of motion (2.5)–(2.6), for the ansatz given in (2.11), (2.17)–(2.19), are equivalent to the equations of motion for a 1-dimensional σ-model with the action (see also [2, 3])

$$S_\sigma = \frac{1}{2} \int du \mathcal{N} \left[G_{ij} \dot\phi^i \dot\phi^j + \dot\varphi^2 + \sum_{I \in \Omega_e} \mathcal{E}(I) \exp[-2U^I(\phi, \varphi)] (\dot\Phi^I)^2 \right], \tag{3.1}$$

the overdots represent differentiation with respect to the distinguished coordinate, i.e. $\frac{d}{du}$.

The factor $\mathcal{N}$ is the lapse function given by

$$\mathcal{N} = \exp(\gamma_0 - \gamma) > 0 \tag{3.2}$$

with the definition

$$\gamma_0(\phi) \equiv \sum_{i=1}^{n} \phi^i \,, \tag{3.3}$$

Next the factor in the exponent is given by

$$U^I = U^I(\phi, \varphi) = -\lambda\varphi + \sum_{i \in I} \phi^i \,. \tag{3.4}$$

Finally,

$$G_{ij} = \delta_{ij} - 1 \tag{3.5}$$

are components of the "pure cosmological" minisupermetric matrix, $i, j = 1, \dots, n$ [16, 17].

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In this rewriting of the system the generalized "Maxwell equations" of (2.6) become

$$\frac{d}{du}\left(\exp(-2U^I)\dot{\Phi}^I\right) = 0\,. \tag{3.6}$$

They can be readily integrated to give

$$\dot{\Phi}^I = Q(I)\exp(2U^I)\,, \tag{3.7}$$

where $Q(I)$ are constant charge densities and $I \in \Omega_e$.

We will analyze the σ-model representation of the original system in the harmonic gauge where

$$\gamma = \gamma_0\,, \qquad \mathcal{N} = 1\,. \tag{3.8}$$

We now further simplify the σ-model in (3.1) by introducing collective variables $x = (x^A) = (\phi^i, \varphi)$ and a "truncated" target space metric

$$\bar{G} = \bar{G}_{AB} dx^A \otimes dx^B = G_{ij} d\phi^i \otimes d\phi^j + d\varphi \otimes d\varphi\,, \tag{3.9}$$

$$(\bar{G}_{AB}) = \begin{pmatrix} G_{ij} & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.10}$$

$U^I(x)$ is defined in (3.4). It can be written as $U^I(x) = U^I_A x^A$

$$(U^I_A) = (\delta_{iI}, -\lambda)\,, \tag{3.11}$$

where

$$\delta_{iI} \equiv \sum_{j\in I} \delta_{ij} = \begin{array}{ll} 1\,, & i \in I\,; \\ 0\,, & i \notin I\,; \end{array} \tag{3.12}$$

is an indicator of i belonging to I. For fixed charge densities $Q(I)$, $I \in \Omega_e$, the equations of motion for the σ-model in (3.1) are now equivalent to the equations from the lagrangian

$$L_Q = \frac{1}{2}\bar{G}_{AB}\dot{x}^A\dot{x}^B - V_Q\,, \tag{3.13}$$

with the zero-energy constraint

$$E_Q = \frac{1}{2}\bar{G}_{AB}\dot{x}^A\dot{x}^B + V_Q = 0\,. \tag{3.14}$$

Here

$$V_Q = \frac{1}{2}\sum_{I\in\Omega_e} \mathcal{E}(I) Q^2(I)\exp[2U^I(x)]\,. \tag{3.15}$$

In section 5 we will examine explicit solutions of the field equations that result from (3.13)–(3.15).

3.2 Constraints

Due to diagonality of the Ricci-tensor for the metric (2.11) the non-diagonal part of the Einstein equations (2.4) reads as follows

$$T_{ij} = 0\,, \qquad i \neq j\,. \tag{3.16}$$

This leads to constraints among the charge densities $Q(I)$. First, the non-diagonal components of stress-energy tensor are proportional to

$$e^{2\lambda\varphi} F_{iM_2\ldots M_q} F_j^{\ M_2\ldots M_q}\,, \tag{3.17}$$

with $i \neq j$. From (2.17), (3.7) and (3.4) we obtain for the $(p+2)$-form

$$F = \frac{1}{(p+1)!} Q_{i_0\ldots i_p} \exp(2\phi^{i_0} + \cdots + 2\phi^{i_p} - 2\lambda\varphi) du \wedge dy^{i_0} \wedge \cdots \wedge dy^{i_p}\,. \tag{3.18}$$

Inserting this in (3.17) we are led to the following constraint equations on charge densities [12]

$$C_{ij} \equiv \sum_{i_1,\ldots,i_p=1}^{n} Q_{ii_1\ldots i_p} Q_{ji_1\ldots i_p} \varepsilon_{i_1} \mathrm{e}^{2\phi^{i_1}} \cdots \varepsilon_{i_p} \mathrm{e}^{2\phi^{i_p}} = 0\,, \tag{3.19}$$

where $i \neq j$; $i,j = 1,\ldots,n$. T_{ij} is proportional to $\exp(-2\lambda\varphi - 2\gamma + 2\phi^i + 2\phi^j) C_{ij}$ for $i \neq j$.

Here $p = q-2$ and $Q_{i_0 i_1 .. i_p}$ are components of the antisymmetric form of rank $p+1 = q-1$ and

$$Q_{i_0 i_1\ldots i_p} = Q(\{i_0, i_1, \ldots, i_p\}) \tag{3.20}$$

for $i_0 < i_1 < \cdots < i_p$ and $\{i_0, i_1, \ldots, i_p\} \in \Omega_e$. The number of constraints in (3.19) is $n(n-1)/2$. In the next section we will show that these constraints can be satisfied when the dimension of space-time takes certain odd values.

4. Solution to constraints on charge densities in various dimensions

The constraints (3.19) can be rewritten as

$$\bar{C}_i^j = \sum_{i_1,\ldots,i_p=1}^{n} \bar{Q}_{ii_1\ldots i_p} \bar{Q}^{ji_1\ldots i_p} = 0\,, \tag{4.1}$$

$i \neq j$; $i,j = 1,\ldots,n$. The charge densities have been redefined via

$$\bar{Q}_{i_0 i_1\ldots i_p} = Q_{i_0 i_1\ldots i_p} \prod_{k=0}^{p} \exp(\phi^{i_k}) \tag{4.2}$$

and the indices were lifted by the flat metric

$$\eta = \varepsilon_1 dy^1 \otimes dy^1 + \cdots + \varepsilon_n dy^n \otimes dy^n = \eta_{ab} dy^a \otimes dy^b\,, \tag{4.3}$$

where $(\eta_{ab}) = (\eta^{ab}) = \mathrm{diag}(\varepsilon_1, \ldots, \varepsilon_n)$, i.e. $\bar{Q}^{i_0\ldots i_p} = \eta^{i_0 i_0'} \cdots \eta^{i_p i_p'} \bar{Q}_{i_0'\ldots i_p'}$. (Here $\bar{C}_i^j = C_i^j \exp(\phi^i + \phi^j)$ with $C_i^j = C_{ik}\eta^{kj}$.)

These $\bar{Q}_{i_0 i_1\ldots i_p}$ can be viewed as "running" charge densities with the functional dependence coming from $\phi^{i_k}(u)$. The charge densities will vary with time or spatially depending on whether u is a time-like or space-like coordinate respectively.

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4.1 Maximal configurations for dimensions $D = 4m + 1$

D=5 case: to help illustrate the preceding general analysis of the constraints in (4.1) we consider the explicit example $D = 5$, $n = 4$ and $\varepsilon_1 = \cdots = \varepsilon_4 = 1$. The constraints of eqs. (4.1) read

$$\bar{Q}_{13}\bar{Q}_{23} + \bar{Q}_{14}\bar{Q}_{24} = 0\,, \tag{4.4}$$
$$\bar{Q}_{12}\bar{Q}_{23} - \bar{Q}_{14}\bar{Q}_{34} = 0\,, \tag{4.5}$$
$$\bar{Q}_{12}\bar{Q}_{24} + \bar{Q}_{13}\bar{Q}_{34} = 0\,, \tag{4.6}$$
$$\bar{Q}_{12}\bar{Q}_{13} + \bar{Q}_{24}\bar{Q}_{34} = 0\,, \tag{4.7}$$
$$\bar{Q}_{12}\bar{Q}_{14} - \bar{Q}_{23}\bar{Q}_{34} = 0\,, \tag{4.8}$$
$$\bar{Q}_{13}\bar{Q}_{14} + \bar{Q}_{23}\bar{Q}_{24} = 0\,. \tag{4.9}$$

It is not difficult to verify that the only non-zero solutions to eqs. (4.4)–(4.9) are

$$\bar{Q}_{12} = \mp\bar{Q}_{34}\,, \tag{4.10}$$
$$\bar{Q}_{13} = \pm\bar{Q}_{24}\,, \tag{4.11}$$
$$\bar{Q}_{14} = \mp\bar{Q}_{23}\,. \tag{4.12}$$

This may be obtained by considering the following three pairs of equations: (i) (4.4) and (4.9); (ii) (4.5) and (4.8); (iii) (4.6) and (4.7). The solution (4.10)–(4.12) may be written in a compact form as

$$\bar{Q}_{i_0 i_1} = \pm\frac{1}{2}\varepsilon_{i_0 i_1 j_0 j_1}\bar{Q}^{j_0 j_1} = \pm(*\bar{Q})_{i_0 i_1}\,, \tag{4.13}$$

where $* = *[\eta]$ is the Hodge operator with respect to η. That means that any self-dual or anti-self-dual 2-form is the solution to a set of quadratic equations (4.4)–(4.9).

D= 4m +1 case: we now look at the general case. Based on the $D = 5$ case we will take the "running" charge density form $\bar{Q}_{i_0 \ldots i_p}$ as self-dual or anti-self-dual in order to satisfy the constraint equations (4.1). This form can only be self-dual or anti-self-dual when the number of the non-distinguished coordinates is twice the rank of the form:

$$n = 2(p + 1)\,. \tag{4.14}$$

Thus, we restrict ourselves to the case when

$$\bar{Q}_{i_0 \ldots i_p} = \pm\frac{1}{(p+1)!}\varepsilon_{i_0 \ldots i_p j_0 \ldots j_p}\bar{Q}^{j_0 \ldots j_p} = \pm(*\bar{Q})_{i_0 \ldots i_p}\,. \tag{4.15}$$

Here the symbol $* = *[\eta]$ is the Hodge operator with respect to η. Squaring the Hodge operator gives

$$(*)^2 = \mathcal{E}(-1)^{(p+1)^2}\mathbf{1}\,, \tag{4.16}$$

where

$$\mathcal{E} = \varepsilon_1 \cdots \varepsilon_n \tag{4.17}$$

equals ± 1 depending on if there are an even or odd number of time-like coordinates.

It can be easily verified that the set of linear equations (4.15) has a non-zero solution if and only if

$$\mathcal{E}(-1)^{(p+1)^2} = 1\,. \tag{4.18}$$

The dimension of the space of solutions is $\frac{1}{2}C^{p+1}_{2(p+1)}$. The factor of $\frac{1}{2}$ comes from (anti-)self-duality condition.

We will now demonstrate how having self dual or anti-self dual charge density form results in the constraints (3.19) being satisfied. First, consider

$$\bar{C}_i^{\ j} = \sum_{i_1,\dots,i_p=1}^{n} \bar{Q}_{ii_1\dots i_p}\bar{Q}^{ji_1\dots i_p} = \sum_{i_1,\dots,i_p=1}^{n}\ \sum_{j_0,\dots,j_p=1}^{n} \pm\frac{1}{(p+1)!}\varepsilon_{ii_1\dots i_p j_0\dots j_p}\bar{Q}^{j_0\dots j_p}\bar{Q}^{ji_1\dots i_p}\,, \tag{4.19}$$

where $i \neq j$, and we have used the requirement of self duality or anti-self duality for the charge density. This can be further rewritten as

$$\begin{aligned} \bar{C}_i^{\ j} &= \sum_{i_1,\dots,i_p=1}^{n}\ \sum_{j_1,\dots,j_p=1}^{n} \pm\frac{1}{p!}\varepsilon_{ii_1\dots i_p j j_1\dots j_p}\bar{Q}^{jj_1\dots j_p}\bar{Q}^{ji_1\dots i_p} \\ &= \sum_{i_1,\dots,i_p=1}^{n}\ \sum_{j_1,\dots,j_p=1}^{n} \pm\frac{(-1)^p}{p!}\varepsilon_{ij_1\dots j_p j i_1\dots i_p}\bar{Q}^{ji_1\dots i_p}\bar{Q}^{jj_1\dots j_p} \\ &= (-1)^p\bar{C}_i^{\ j}\,. \end{aligned} \tag{4.20}$$

Note that j is not summed over in the two sums above. In going from the second line of (4.19) to the first line of (4.20) we have carried out $p+1$ identical sums with: $j_0 = j$, $j_1 = j, \dots, j_p = j$, respectively.

From (4.20) one finds that the constraints in (4.1) are satisfied automatically for odd $p = 2m-1$ since

$$\bar{C}_i^{\ j} = -\bar{C}_i^{\ j} \Rightarrow \bar{C}_i^{\ j} = 0\,.$$

From (4.18) we get for odd p

$$\mathcal{E} = 1\,, \tag{4.21}$$

i.e. the metric (4.3) is either euclidean or has an even number of time-like directions.

The relationship between n and p is $n = 2(p+1)$. Thus, we find non-trivial solutions to the constraints (4.1) when the total spacetime dimension is

$$D = n+1 = 4m+1 = 5, 9, 13, \dots \tag{4.22}$$

and the signature parameter $\mathcal{E}$ is positive.

4.2 Absence of maximal configurations for $p = 1$ and even D

In this subsection we show that for the even dimensional case ($D = 2k$) with 3-form ($p = 1$) there are no solutions with maximal number of electric $S1$-branes.

Here we put $\varepsilon_1 = \dots = \varepsilon_n = 1$. Equations (4.1) imply in this case

$$\sum_{i_1=1}^{n} \bar{Q}_{ii_1}\bar{Q}_{ji_1} = \delta_{ij}P_i \equiv P_{ij}\,. \tag{4.23}$$

The indices are not summed in the second term. Now we assume that all $\bar{Q}_{ij} \neq 0$, $i \neq j$, i.e. a composite 1-brane configuration with maximal number of electric branes ("strings") is considered, and show that this leads to an inconsistency. The constants P_i, which are the values of an $n \times n$ diagonal matrix, satisfy $P_i > 0$, since when $i = j$ the first equation is just a sum of squares. The exact values of P_i's will not be needed here. In matrix notation (4.23) reads

$$-\bar{Q}^2 = P\,, \tag{4.24}$$

where we have used the antisymmetry relation $\bar{Q}_{ij} = -\bar{Q}_{ji}$. Calculation of the determinants of the matrices in the previous relation leads to

$$(-1)^n (\det\bar{Q})^2 = \det P > 0\,. \tag{4.25}$$

which is not valid for odd n. (For odd n $\det\bar{Q} = 0$.) Thus, there are no "maximal" solutions to constraints for even dimensions D and $p = 1$ in the model under consideration.

This implies the absence of "maximal" configurations of composite electric $S1$-branes in 10-dimensional supergravities and low-energy models of superstring origin when only one 3-form is considered.

In the next section we examine the cosmological type solutions to the field equations for $D = 5, 9, 13, \ldots$ with the maximal number of non-zero charge densities $Q_{j_0 j_1 \ldots j_p}$ obeying (4.15).

5. Cosmological solutions to the field equations for $D = 5, 9, 13, \ldots$

Here we give explicit examples of cosmological type solutions for the dimensions from (4.22) when the non-distinguished coordinates are all space-like, i.e.

$$\varepsilon_1 = \cdots = \varepsilon_n = 1\,. \tag{5.1}$$

First we show that all scale factors are the same up to constants:

$$\phi^i(u) = \phi(u) + c^i\,. \tag{5.2}$$

From the definition of running constants (4.2) and the (anti-) self-duality of the charge density form (4.15), it follows that

$$\sum_{i \in I} \phi^i = \sum_{j \in \bar{I}} \phi^j + \mathrm{const}\,, \tag{5.3}$$

where I is an arbitrary subset of $I_0 = \{1, \ldots, n\}$ of length $n/2 = 2m$ and

$$\bar{I} \equiv I_0 \setminus I\,, \tag{5.4}$$

is "dual" set. For $D = 5$ case eqs. (5.3) read: $\phi^1 + \phi^2 = \phi^3 + \phi^4 + \mathrm{const}$ and two other relations obtained by permutations.

From relations (5.3) one can see that all ϕ^i should coincide up to constants (for $i \in I$ and $j \in \bar{I}$ it is sufficient to consider another equation with $I_1 = (I \setminus \{i\}) \cup \{j\})$ instead of I in (5.3) and find from both equations that ϕ^i and ϕ^j coincide up to a constant).

Thus, we are led to (5.2). In what follows we put $c^i = 0$ which may always be done via a proper rescaling of y-coordinates. This also implies that non-running charge density form $Q_{i_0 \dots i_p}$ is self-dual or anti-self-dual in a flat euclidean space $\mathbb{R}^n$, i.e.

$$Q_{i_0 \dots i_p} = \pm \frac{1}{(p+1)!} \varepsilon_{i_0 \dots i_p j_0 \dots j_p} Q^{j_0 \dots j_p} = \pm (*Q)_{i_0 \dots i_p} \,. \tag{5.5}$$

The lagrangian and total energy constraint are given by

$$L_Q = \frac{1}{2} G_{ij} \dot{\phi}^i \dot{\phi}^j - V_Q + \frac{1}{2} \dot{\varphi}^2 \,, \tag{5.6}$$

$$E_Q = \frac{1}{2} G_{ij} \dot{\phi}^i \dot{\phi}^j + V_Q + \frac{1}{2} \dot{\varphi}^2 = 0 \,, \tag{5.7}$$

with the potential being

$$V_Q = \frac{1}{2} \sum_I Q^2(I) \exp\left(2 \sum_{k \in I} \phi^k - 2\lambda\varphi \right), \tag{5.8}$$

see (3.13)–(3.15).

The field equations for ϕ and φ from L_Q are

$$\sum_{j=1}^{n} G_{ij} \ddot{\phi}^j + \sum_I Q^2(I) \delta_I^i \exp\left(2 \sum_{k \in I} \phi^k - 2\lambda\varphi \right) = 0 \,, \tag{5.9}$$

$$\ddot{\varphi} + \sum_I Q^2(I)(-\lambda) \exp\left(2 \sum_{k \in I} \phi^k - 2\lambda\varphi \right) = 0 \,. \tag{5.10}$$

Since all ϕ^i satisfy $\phi^i = \phi$ one finds $\sum_{j=1}^n G_{ij} \ddot{\phi}^j = \sum_{j=1}^n (\delta_{ij} - 1) \ddot{\phi}^j = (1-n)\ddot{\phi}$. Next, defining

$$Q^2 \equiv \sum_I Q^2(I) \neq 0 \tag{5.11}$$

and noting that

$$\sum_I Q^2(I) \delta_I^i = \frac{1}{2} Q^2 \tag{5.12}$$

for any $i = 1, \dots, n$, one finds that the field equations (5.9) and (5.10) become

$$\ddot{\phi} = \frac{1}{2(n-1)} Q^2 \exp(n\phi - 2\lambda\varphi) \,, \tag{5.13}$$

$$\ddot{\varphi} = \lambda Q^2 \exp(n\phi - 2\lambda\varphi) \,. \tag{5.14}$$

Finally, since the exponents in (5.13) and (5.14) are the same these two equations can be combined into one as

$$\ddot{f} = -2Ae^{2f} \,, \tag{5.15}$$

with the definitions

$$f \equiv \frac{n}{2} \phi - \lambda\varphi \,, \tag{5.16}$$

$$A \equiv \frac{Q^2}{2} K \,, \qquad K \equiv \lambda^2 - \frac{n}{4(n-1)} \,, \tag{5.17}$$

assumed.

The first integral of (5.15) is

$$\frac{1}{2}\dot{f}^2 + Ae^{2f} = \frac{1}{2}C\,, \tag{5.18}$$

where C is an integration constant.

Let $K \neq 0$, or

$$\lambda^2 \neq \frac{n}{4(n-1)} \equiv \lambda_0^2\,. \tag{5.19}$$

Equation (5.15) has several solutions:

$$f = -\ln\left[z|2A|^{1/2}\right] \tag{5.20}$$

with

$$z = \frac{1}{\sqrt{C}}\sinh\left[(u-u_0)\sqrt{C}\right], \qquad A<0\,, \quad C>0\,; \tag{5.21}$$

$$= \frac{1}{\sqrt{-C}}\sin\left[(u-u_0)\sqrt{-C}\right], \qquad A<0\,, \quad C<0\,; \tag{5.22}$$

$$= u-u_0\,, \qquad A<0\,, \quad C=0\,; \tag{5.23}$$

$$= \frac{1}{\sqrt{C}}\cosh\left[(u-u_0)\sqrt{C}\right], \qquad A>0\,, \quad C>0\,. \tag{5.24}$$

One can relate the solutions given in (5.20) and (5.21)–(5.24) to ϕ and φ by using (5.13) to construct the following relationship

$$\ddot{\varphi} = 2(n-1)\lambda\ddot{\phi}\,, \tag{5.25}$$

which has the solution

$$\varphi = 2(n-1)\lambda\phi + C_2 u + C_1\,, \tag{5.26}$$

where C_2, C_1 are integration constants. Combining (5.26) with (5.16) gives

$$\phi = \frac{1}{2(1-n)K}\left[\lambda(C_2 u + C_1) + f(u)\right], \qquad \varphi = \frac{n}{4(1-n)K}(C_2 u + C_1) - \frac{\lambda f(u)}{K}\,. \tag{5.27}$$

Applying this to the zero energy constraint

$$E_Q = \frac{1}{2}n(1-n)\dot{\phi}^2 + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}Q^2 e^{2f(u)} = 0 \tag{5.28}$$

and using (5.18) we get

$$E_Q = \frac{C}{K} - \frac{n(C_2)^2}{4K(n-1)} = 0\,, \tag{5.29}$$

or equivalently,

$$C = \frac{n}{4(n-1)}(C_2)^2 \geq 0\,. \tag{5.30}$$

This tells us that only the three cases (5.21), (5.23) and (5.24) occur when real solutions are considered. The solution (5.22) will be considered in a possible future work with a pure imaginary scalar field and λ (this is equivalent to a "phantom" field that may support the so-called bouncing solution).

Collecting these results together the solutions for the metric, scalar field and $(p+2)$-form are

$$ds^2 = we^{2n\phi(u)}du^2 + e^{2\phi(u)}\sum_{i=1}^{n}(dy^i)^2 \tag{5.31}$$

$$\varphi = \frac{n}{4(1-n)K}\,(C_2 u + C_1) - \frac{\lambda f(u)}{K}\,, \tag{5.32}$$

$$F = e^{2f(u)}du \wedge Q\,, \qquad Q = \frac{1}{(p+1)!}Q_{i_0\ldots i_p}dy^{i_0}\wedge\cdots\wedge dy^{i_p}\,, \tag{5.33}$$

with $\phi(u)$ given by (5.27) and the function $f(u)$ given by (5.20) and (5.21), (5.23), (5.24). Here the charge density form Q of rank $n/2 = 2m$ is self-dual or anti-self-dual in a flat euclidean space $\mathbb{R}^n$: $Q = \pm{*Q}$, the parameters C_2, C obey (5.30) and the dilatonic coupling constant λ is non-exceptional, see (5.19).

5.1 Special attractor solution for $C = 0$

Here we examine some properties of the simplest cosmological type solution given in (5.23). For this solution $A < 0$ and hence

$$\lambda^2 < \lambda_0^2\,. \tag{5.34}$$

Since $C = 0$ for (5.23) one has $C_2 = 0$ from the condition (5.30). Finally, without loss of generality we put $u_0 = 0$.

To get a physical understanding of this solution one should change the "time" coordinate u to the proper time coordinate τ. In order to get the correct sign for the proper time we fix the sign in the relationship between u and the proper time as

$$d\tau = -e^{n\phi(u)}du\,, \tag{5.35}$$

where

$$\phi = \frac{1}{2(n-1)K}\ln(u|2\bar{A}|^{1/2})\,, \qquad \bar{A} = Ae^{-2\lambda C_1}\,. \tag{5.36}$$

Integrating (5.35) and taking a suitable choice of reference point we get

$$|\alpha||2\bar{A}|^{1/2}\tau = (u|2\bar{A}|^{1/2})^{\alpha}\,, \tag{5.37}$$

where $u > 0$ and

$$\alpha = \frac{\lambda^2+\lambda_0^2}{\lambda^2-\lambda_0^2} < 0\,. \tag{5.38}$$

Since $\alpha < 0$, $\tau = \tau(u)$ is monotonically decreasing from infinity when $u = 0$ to zero when $u = \infty$.

The metric (5.31) now reads

$$ds^2 = wd\tau^2 + B\tau^{2\nu}\sum_{i=1}^{n}(dy^i)^2\,, \tag{5.39}$$

where $\tau > 0$ and

$$\nu = \frac{2}{n+4\lambda^2(n-1)}\,, \qquad B = (|\alpha||2\bar{A}|^{1/2})^{2\nu}\,. \tag{5.40}$$

By putting $\lambda = 0$ and $w = -1$ in the above solution we get a cosmological power-law expansion with a power $\nu = 2/n$ that is the same as in the case of $D = 1 + n$ dust matter with a zero pressure (see for example [18, 19, 16]). This is not surprising since it can be argued that the collection of branes with charge densities obeying (anti)-self-duality condition (5.5) behaves as a dust matter. Indeed, we know that in the absence of a scalar field the solution with a single brane described by a set $I \subset \{1, \dots, n\}$ is equivalent to an anisotropic fluid with equations of state $p_i = -\rho$ for $i \in I$ and $p_i = \rho$ otherwise, $i = 1, \dots, n$ [5]. Here p_i is a pressure in the i-th direction and ρ is the energy density. Thus, our collection of branes is equivalent to a multicomponent fluid [20] with coinciding densities, since all $Q^2(I)$ are equal and $\phi^i = \phi$. For any i the collection of branes can be split into pairs with sets $I, \bar{I}$ such that $i \in I$. The first brane gives pressure $p_i^I = -\rho$ and the second one gives $p_i^{\bar{I}} = \rho$ (energy densities are the same). Hence the total pressure is zero for the pair and, thus, for the whole collection of branes.

Finally, we note that the solution (5.39) is attractor solution in the limit $\tau \to +\infty$, or $u \to +0$, for the solutions with $A < 0$ given by (5.21). This follows just from relation $\sinh u \sim u$ for small u.

5.2 Kasner-like behavior for $\tau \to +0$

We now consider $u \to +\infty$ asymptotical behavior of solutions with i) sinh- and ii) cosh-functions corresponding to (5.21) and (5.24), respectively. In both cases we find that the asymptotic behavior is Kasner like i.e. the metric and scalar field take the form

$$ds_{as}^2 = w d\tau^2 + \sum_{i=1}^{n} \tau^{2\alpha^i} A_i (dy^i)^2 , \qquad \varphi_{as} = \alpha_\varphi \ln \tau + \varphi_0 , \tag{5.41}$$

where $A_i > 0$, φ_0 are constants. The Kasner parameters obey

$$\sum_{i=1}^{n} \alpha^i = \sum_{i=1}^{n} (\alpha^i)^2 + (\alpha_\varphi)^2 = 1 . \tag{5.42}$$

In the first case i) with sinh-dependence and $A < 0$ the proper time τ is decreasing when $u \to +\infty$. From eqs. (5.20), (5.21), (5.24), (5.27) and (5.35) we find the following asymptotic behavior as $u \to +\infty$

$$\phi \sim -bu + \text{const} , \qquad b = \frac{\lambda C_2 - \sqrt{C}}{2(n-1)K} > 0 , \tag{5.43}$$

$$\varphi \sim (-\lambda_0^2 C_2 + \lambda \sqrt{C}) K^{-1} u + \text{const} , \tag{5.44}$$

$$\tau \sim \text{const} \exp(-nbu) . \tag{5.45}$$

Using these asymptotic relations and writing everything in terms of proper time one find that the metric and scalar field take the following asymptotic forms

$$ds_{as}^2 = w d\tau^2 + \tau^{2/n} A_0 \sum_{i=1}^{n} (dy^i)^2 , \qquad \varphi_{as} = \alpha_\varphi \ln \tau + \varphi_0 , \tag{5.46}$$

$$\alpha_\varphi^2 = 1 - \frac{1}{n} , \tag{5.47}$$

as $\tau \to +0$. Here $A_0 > 0$, φ_0 are constants. The relationship for α_φ^2 comes from (5.44), (5.45) and agrees with (5.42) and $\alpha^i = 1/n$. Using (5.43)–(5.45) one can obtain the following relationship: $\mathrm{sign}[\alpha_\varphi] = -\mathrm{sign}[C_2]$.

In the second case ii) with cosh-dependence and $A > 0$ the proper time τ decreases as $u \to +\infty$ for $\lambda C_2 > 0$ and increases for $\lambda C_2 < 0$. In this case we also get an asymptotical Kasner type relations (5.46)–(5.47) in the limit $\tau \to +0$ with $\mathrm{sign}[\alpha_\varphi] = -\mathrm{sign}[\lambda]$. In both sinh$-$ and cosh$-$ cases the Kasner sets $\alpha = (\alpha^i = 1/n, \alpha_\varphi)$ obey the inequalities

$$U^I(\alpha) = -\lambda\alpha_\varphi + \sum_{i \in I} \alpha^i = -\lambda\alpha_\varphi + \frac{1}{2} > 0 \tag{5.48}$$

for all brane sets I. This is in agreement with a general prescription of the billiard representation from [13]. In the case ii) we obtain the Kasner type behavior (5.46)–(5.47) in the limit $\tau \to +\infty$ with $\mathrm{sign}[\alpha_\varphi] = \mathrm{sign}[\lambda]$. In this case

$$U^I(\alpha) < 0 \tag{5.49}$$

for all I. In case ii) the scalar field has a bouncing behavior in the interval $\tau \in (0, +\infty)$.

6. Conclusions

In this article we have examined a system where $D = n+1$ dimensional gravity was coupled to a scalar field plus a $p+2$ rank form field. The ansatz employed here was to take the metric as diagonal, and the rank $p+2$ form field to have a composite electric Sp-brane form. All ansatz functions depended only on the one distinguished coordinate, u. Under these conditions the initial model could be reduced to an effective 1-dimensional σ-model which greatly simplified the study of the system. The diagonal character of our metric ansatz resulted in there being constraint equations among the charge densities of branes associated with the $p+2$ form field. By examining these constraint equations we showed that for certain odd values of the spacetime dimension, given by $D = 4m+1 = 5, 9, 13, \ldots$, the system allowed the maximal number of the charged branes. We also proved the absence of such maximal configurations for $p = 1$ and even D.

For these special odd dimensions and non-exceptional dilatonic coupling ($\lambda^2 \neq \frac{n}{4(n-1)}$) we wrote down exact solutions given in equations (5.31)–(5.33). We examined the simplest of these solutions given by (5.23). On converting the distinguished coordinate u to the proper time τ, we found this solution corresponded to a cosmological power-law expansion. By taking the limit of vanishing dilatonic coupling, $\lambda = 0$, we showed that this solution reduced to a cosmological model with dust matter. On physical grounds this is exactly what one would expect in this limit. We also investigated an asymptotical Kasner type behavior of the solutions for small ($\tau \to +0$) and large ($\tau \to +\infty$) values of proper time.

Directions for possible future work include: (i) the investigation of the properties of the solution in (5.22) using pure imaginary scalar field and λ (with a hope to investigate the possibility of bouncing behavior); (ii) the investigation of static, non-cosmological solutions, when $w = +1$ and the distinguished coordinate u is spacelike; (iii) examining the exceptional case, when $K = 0$ (i.e. $\lambda^2 = \frac{n}{4(n-1)}$) in (5.17).

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Using the results obtained in this paper and in [13] we will present in future work [29] some examples of breaking of the never-ending oscillating behavior near the singularity by constraints. Physically this may indicate that the form of the ansatz considered here (diagonal metric, composite electric ansatz for the form field, *etc.*) may prevent the occurrence of the "chaotic", never-ending oscillating behavior (see [13, 14, 15, 21, 22] and references, therein).

It should be noted that recently N.S. Deger has obtained in [23] some S-brane solutions in $D = 11$ supergravity with non-standard intersection rules. These solutions contain "dangerous" intersections of branes and give certain examples of solutions with charge densities obeying constraint equations. It seems that the "non-standard" solutions from [23] with vanishing Chern-Simons term may be obtained as a special case of the so-called "block-orthogonal" solutions [24, 25, 26]. In these block-orthogonal solutions the branes split into several blocks. The branes belonging to one block behave as one "effective" brane. The branes belonging to different blocks obey the "standard" intersection rules [12, 27, 28]. In future work [29] we will verify whether the solution from our paper may be obtained using the modified one-block solution from [25, 26].

Acknowledgments

The work of V.D.I. was supported in part by a DFG grant. The work of D.S. was supported by a 2003-2004 Fulbright Fellowship. D.S. would like to thank Vitaly Melnikov and Michael Fil'chenkov for their invitation to visit and support during his stay at PFUR.

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